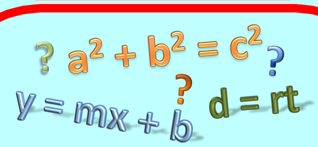
Math 115

Spring 2019

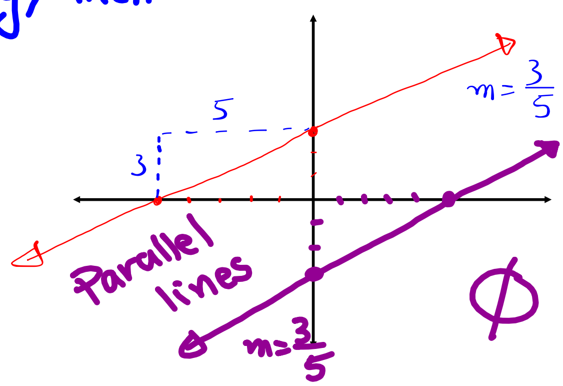
Lecture 19



Exam/Quiz

① Solve by graphing, then discuss system and equations.

$$\begin{cases} 3x - 5y = -15 \\ y = \frac{3}{5}x - 3 \end{cases}$$



System: Inconsistent

Eqns: Independent

② Solve, then discuss system and equations

$$2\begin{cases} 2x - 7y = 10 \\ -4x + 14y = -20 \end{cases} \rightarrow \begin{cases} \cancel{4x} - \cancel{14y} = 20 \\ -\cancel{4x} + \cancel{14y} = -20 \end{cases}$$

$$0 = 0$$

→ True

infinitely many Solns.

System = Consistent

Eqns: Dependent

Some Review:

1) Evaluate $\frac{x^3+8}{x+2}$ for $x=0$, $x=-2$.

for $x=0$

$$\frac{0^3+8}{0+2}=\frac{0+8}{2}=\boxed{4}$$

for $x=-2$

$$\frac{(-2)^3+8}{-2+2}=\frac{-8+8}{-2+2}=\frac{0}{0}$$

$\boxed{\text{Indeterminate}}$

2) Simplify: $3(2x^2-5x+4)-5x-12$

$$=6x^2-15x+\cancel{12}-5x-\cancel{12}$$

$$=\boxed{6x^2-20x}$$

Binomial
D=2
L.C. 6
No constant

3) Use exponential rules to simplify

a) $(-4x^5)^3\cdot x^7$

Monomial
D=22
C=-64

$$=(-4)^3(x^5)^3\cdot x^7$$

$$=-64x^{15}x^7=\boxed{-64x^{22}}$$

b) $\frac{\cancel{-48}^{\,-16}x^{12}y^3}{\cancel{15}_{\,5}x^2y^{10}}$

$$=\boxed{\frac{-16x^{10}}{5y^7}}$$

Expression
Not
Monomial

c) $\left(\frac{2x^4}{y^6}\right)^{-3}$

$$=\left(\frac{y^6}{2x^4}\right)^3=\boxed{\frac{y^{18}}{8x^{12}}}$$

Not a monomial

d) $\frac{(x^6)^8\cdot(x^4)^2}{x^{50}\cdot x^5}$

$$=\frac{x^{48}\cdot x^8}{x^{50+5}}=\frac{x^{56}}{x^{55}}=\boxed{x}$$

Monomial

Simplify:

a) $(8.2 \times 10^{-12})(4.5 \times 10^{-8})$

$= 36.9 \times 10^{-20}$

$= 3.69 \times 10^{1} \times 10^{-20}$

$= \boxed{3.69 \times 10^{-19}}$

b) $\dfrac{7.5 \times 10^{-13}}{1.5 \times 10^{7}}$

$= \boxed{5 \times 10^{-20}}$

Use foil to multiply

a) $(3x^4 - 5x^2)(2x^4 + 3x^2)$

$= 6x^8 + 9x^6 - 10x^6 - 15x^4$

$= \boxed{6x^8 - x^6 - 15x^4}$

Trinomial
D=8, L.C.=6
No const.

b) $(4x-3)(16x^2+12x+9)$

$= 64x^3 + \cancel{48x^2} + \cancel{36x}$
$- \cancel{48x^2} - \cancel{36x} - 27$

$= \boxed{64x^3 - 27}$

const.=-27

Binomial, D=3, L.C.=64

Use special products

$(A+B)^2 = A^2 + 2AB + B^2$

$(A-B)^2 = A^2 - 2AB + B^2$

$(A+B)(A-B) = A^2 - B^2$

to find

a) $(3x^4 + 2y^5)^2$

$= (3x^4)^2 + 2(3x^4)(2y^5) + (2y^5)^2$

$= \boxed{9x^8 + 12x^4y^5 + 4y^{10}}$

Trinomial
D=10
LC=4

b) $(7x^3 - 3x^2)^2$

$= (7x^3)^2 - 2(7x^3)(3x^2) + (3x^2)^2$

$= \boxed{49x^6 - 42x^5 + 9x^4}$

Trinomial, D=6,
L.C.=49,
No Constant

c) $\underbrace{(5x^7 - 9)(5x^7 + 9)}_{\text{Conjugates}} = (5x^7)^2 - (9)^2$

$= \boxed{25x^{14} - 81}$

Binomial, D=14, L.C.=25, Const=-81

Find an expression in simplest form for the perimeter and area:

$P = 2L + 2W$

$A = LW$

x^2-2x+1

x^2+2x+1

$P = 2L + 2W = 2(x^2+2x+1) + 2(x^2-2x+1)$

$= 2x^2 + \cancel{4x} + 2 + 2x^2 - \cancel{4x} + 2 = \boxed{4x^2+4}$ Binomial

$D=2$, $LC=4$, $Const=4$

$A = LW = (x^2+2x+1)(x^2-2x+1)$

$= x^4 \cancel{-2x^3} + x^2 + \cancel{2x^3} - 4x^2 \cancel{+2x} + x^2 \cancel{-2x} + 1$

$= \boxed{x^4 - 2x^2 + 1}$ Trinomial, $D=4$, $LC=1$, $Const=1$

Division with polynomials:

a) $\dfrac{\text{Polynomial}}{\text{Monomial}}$

$\dfrac{\cancel{25}^{5} x^5}{\cancel{5}_{1} x^2} = 5x^{5-2} = \boxed{5x^3}$ Monomial

$D=3$, $C=5$

$\dfrac{30x^7 - 12x^4}{6x^3} = \dfrac{\cancel{30}^{5} x^7}{\cancel{6} x^3} - \dfrac{\cancel{12}^{2} x^4}{\cancel{6} x^3}$

$= 5x^{7-3} - 2x^{4-3}$

$= \boxed{5x^2 - 2x}$ Binomial

$D=2$, $LC=5$

$$\frac{45x^8y^3 + 18x^6y^2 - 27x^4y}{9x^3y}$$

$$= \frac{\overset{5}{\cancel{45}}x^8y^3}{\cancel{9}x^3y} + \frac{\overset{2}{\cancel{18}}x^6y^2}{\cancel{9}x^3y} - \frac{\overset{3}{\cancel{27}}x^4\cancel{y}}{\cancel{9}x^3\cancel{y}}$$

$$= \boxed{5x^5y^2 + 2x^3y - 3x}$$

Trinomial
D=7
L.C.=5

D=7, C=5 D=4, C=2 D=1, C=−3

Divide $\dfrac{20x^8 - 16x^6 + 4x^3}{-4x^3}$

$$= \frac{20x^8}{-4x^3} - \frac{16x^6}{-4x^3} + \frac{4x^3}{-4x^3}$$

$$= \boxed{-5x^5 + 4x^3 - 1}$$

Trinomial, D=5, LC=−5,
Constant = −1

$\dfrac{\text{Polynomial}}{\text{Polynomial}} \Rightarrow$ Long Division

$$\frac{x^3 + 2x^2 - 5x + 2}{x - 1}$$

$x\,\boxed{x^2} = x^3$

$x\,\boxed{3x} = 3x^2$

$x\,\boxed{-2} = -2x$

$$
\begin{array}{r|llll}
 & x^2 & +3x & -2 & \\
\hline
x-1 & x^3 & +2x^2 & -5x & +2 \\
 & -(x^3 & -x^2) & & \\
\hline
 & & 3x^2 & -5x & +2 \\
 & & -(3x^2 & -3x) & \\
\hline
 & & & -2x & +2 \\
 & & & -(-2x & +2) \\
\hline
\end{array}
$$

$x^2 + 3x - 2$

Remainder $\rightarrow 0$

Divide $\dfrac{3x^2+5x-7}{x+2}$

$x\boxed{3x} = 3x^2$

$x\boxed{-1} = -x$

$$
\begin{array}{r|rrr}
 & 3x & -1 & \\
\hline
x+2 & 3x^2 & +5x & -7 \\
 & -(3x^2 & +6x &) \\
\hline
 & & -x & -7 \\
 & & -(-x & -2) \\
\hline
\end{array}
$$

Remainder $\rightarrow -5$

$3x-1+\dfrac{-5}{x+2}$

Always $+$

Divide

$\dfrac{3x^2-5x+4x^3-8}{x-3}$ $\rightarrow$ has to be in descending order

$x\boxed{4x^2} = 4x^3$

$x\boxed{15x} = 15x^2$

$x\boxed{40} = 40x$

$$
\begin{array}{r|rrrr}
 & 4x^2 & +15x & +40 & \\
\hline
x-3 & 4x^3 & +3x^2 & -5x & -8 \\
 & -(4x^3 & -12x^2 & &) \\
\hline
 & & 15x^2 & -5x & -8 \\
 & & -(15x^2 & -45x &) \\
\hline
 & & & 40x & -8 \\
 & & & -(40x & -120) \\
\hline
\end{array}
$$

Remainder $\rightarrow 112$

$4x^2+15x+40+\dfrac{112}{x-3}$

Divide

$$\frac{6x^3+1-2x-5x^2}{1+2x} = \frac{6x^3-5x^2-2x+1}{2x+1}$$

Hint: Take care of orders first

$2x\,\boxed{3x^2} = 6x^3$

$2x\,\boxed{-4x} = -8x^2$

$2x\,\boxed{1} = 2x$

$$\begin{array}{r|rrrr} & 3x^2 & -4x & +1 & \\ \hline 2x+1 & 6x^3 & -5x^2 & -2x & +1 \\ & -(6x^3 & +3x^2) & & \\ \hline & & -8x^2 & -2x & +1 \\ & & -(-8x^2 & -4x) & \\ \hline & & & 2x & +1 \\ & & & -(2x & +1) \\ \hline \end{array}$$

Remainder $\rightarrow 0$

$3x^2-4x+1$

Divide $\frac{x^3+4x^2-6}{x+1}$ missing term

$$= \frac{x^3+4x^2+0x-6}{x+1}$$

$x\,\boxed{x^2} = x^3$

$x\,\boxed{3x} = 3x^2$

$x\,\boxed{-3} = -3x$

$$\begin{array}{r|rrrr} & x^2 & +3x & -3 & \\ \hline x+1 & x^3 & +4x^2 & +0x & -6 \\ & -(x^3 & +x^2) & & \\ \hline & & 3x^2 & +0x & -6 \\ & & -(3x^2 & +3x) & \\ \hline & & & -3x & -6 \\ & & & -(-3x & -3) \\ \hline \end{array}$$

Remainder $\rightarrow -3$

$x^2+3x-3+\frac{-3}{x+1}$

Divide: $\dfrac{8x^3 + 30}{2x + 3} = \dfrac{8x^3 + 0x^2 + 0x + 30}{2x + 3}$

$2x\,\boxed{4x^2} = 8x^3$

$2x\,\boxed{-6x} = -12x^2$

$2x\,\boxed{+9} = 18x$

$$
\begin{array}{r}
4x^2 \quad -6x \quad +9 \\
2x+3\,\overline{)\,8x^3 + 0x^2 + 0x + 30} \\
-(8x^3 + 12x^2 \qquad\qquad) \\
\hline
-12x^2 + 0x + 30 \\
-(-12x^2 - 18x \quad) \\
\hline
18x + 30 \\
-(18x + 27) \\
\hline
\end{array}
$$

Remainder $\rightarrow 3$

$4x^2 - 6x + 9 + \dfrac{3}{2x+3}$

Divide:

$\dfrac{x^4 - 5x^2 - 36}{x^2 + 5}$

$$
\begin{array}{r}
x^2 \qquad\qquad -10 \\
x^2+5\,\overline{)\,x^4 + 0x^3 - 5x^2 + 0x - 36} \\
-(x^4 \qquad + 5x^2 \qquad\qquad) \\
\hline
-10x^2 + 0x - 36 \\
-(-10x^2 \qquad - 50) \\
\hline
14
\end{array}
$$

$x^2\,\boxed{x^2} = x^4$

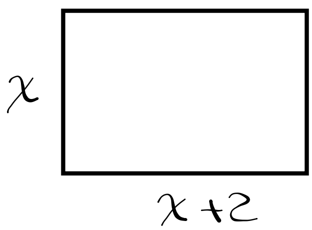
$x^2\,\boxed{-10} = -10x^2$

$x^2 - 10 + \dfrac{14}{x^2+5}$

The dimensions of a rectangular shape are two cons. odd integers.

Find area & Perimeter

Binomial
$D=1$, $LC=4$
Const$=4$



$P = 2L + 2W$

$= 2(x+2) + 2(x)$

$= 2x + 4 + 2x = \boxed{4x+4}$

$A = LW$

$= x(x+2)$

$= \boxed{x^2 + 2x}$

Binomial
$D=2$
$LC=1$
No Constant

The area of a rectangular shape is $4x^2-9$.

Its length is $2x+3$. Find its width.

$A = LW$

$W = \dfrac{A}{L} = \dfrac{4x^2-9}{2x+3}$

$2x\,\boxed{2x} = 4x^2$

$2x\,\boxed{-3} = -6x$

$$\begin{array}{r|l} & 2x \quad -3 \\ 2x+3 & 4x^2 + 0x - 9 \\ & -(4x^2 + 6x) \\ \hline & -6x - 9 \\ & -(-6x - 9) \\ \hline & 0 \end{array}$$

$2x - 3$

Find the width.

$A = 3x^3 - x^2 + 2x + 20$

$3x + 5$

$3x \boxed{x^2} = 3x^3$

$3x \boxed{-2x} = -6x^2$

$3x \boxed{+4} = 12x$

Review Last example

$$W = \frac{A}{L}$$

$$= \frac{3x^3 - x^2 + 2x + 20}{3x + 5}$$

$$\begin{array}{r} x^2 \quad -2x \quad +4 \\ 3x+5 \overline{\big) \; 3x^3 \quad -x^2 \quad +2x \quad +20} \\ -(3x^3 \quad +5x^2 \qquad\qquad\qquad) \\ \hline -6x^2 \quad +2x + 20 \\ -(-6x^2 \quad -10x \qquad) \\ \hline 12x + 20 \\ -(12x + 20) \\ \hline 0 \end{array}$$

width.

$x^2 - 2x + 4$

Due Monday: SG 14

Due Tuesday: Project II
Working with Polynomials